LS-134
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THEORETICAL ESTIMATION OF THE DYNAMIC APERTURE FOR A CHASMAN-GREEN LATTICE

If ν_x is close enough to $\nu_x = 1$ per period, the Hamiltonian for the particle motion in a lattice with reflective symmetry can be written as

$$H = \sqrt{8\, J_x}\, [J_x A_{33} \cos Q_{33} + (3\, J_x A_{11} - 6\, J_y\, B_{11}) \cos Q_{11}]$$

where $Q_{jm} = j\,(\phi_x - \psi_x) + (j\nu_x - m)\theta$

ϕ_x is the particle phase,

ψ_x is the lattice phase,

and θ is the angular coordinate along the orbit.

Using a reflective symmetry point as reference, the harmonic components are given by

$$A_{jm} = \sum_k \frac{S_k}{48\pi} \cos \left(j\psi_x - (j\nu_x - m)\theta\right)$$

$$B_{11} = \sum_k \frac{\bar{S}_k}{48\pi} \cos \left(\psi_x - (\nu_x - 1)\theta\right)$$

where $$S_k = \left(\frac{\beta_x^{3/2} B''(x)\, \ell}{B\rho}\right)_k ,$$

$$\bar{S}_k = \left(\frac{\beta_x^{1/2} \beta_y B''(x)\, \ell}{B\rho}\right)_k ,$$

and $\frac{B''(x)\,\ell}{B\rho}$ are the sextupole strengths.

Since

$$H'(\theta) = -\sqrt{8}\,(\nu_x - 1)\,J_x^{1/2}[3\,A_{33}J_x \sin Q_{33} + (3\,J_x A_{11} - 6\,J_y B_{11})\sin Q_{11}]$$

and

$$J'(\theta) = \sqrt{8}\,J_x^{1/2}[3\,A_{33}J_x \sin Q_{33} + (3\,J_x A_{11} - 6\,J_y B_{11})\sin Q_{11}]$$

the function

$$(\nu_x - 1)\,J_x + H = C \text{ (a constant)}$$

$(J_y$ is also a constant)

We define

$$\Delta = \nu_x - 1\ ,$$

$$N_x = \sqrt{\frac{2J_x}{\varepsilon}}\ ,$$

$$N_y = \sqrt{\frac{4\,J_y}{\varepsilon}}\ , \text{ (constant)}$$

where ε is the natural emittance.

In these variables

$$\frac{\Delta\varepsilon\,N_x^2}{2} + \varepsilon^{3/2} N_x\,[N_x^2\,A_{33} \cos Q_{33} + 3(N_x^2\,A_{11} - N_y^2 B_{11})\cos Q_{11}] = C$$

Define further

$$X_o = \frac{A}{2\sqrt{\varepsilon}A}\ ,$$

$$D = \frac{3\,B_{11}}{A}\ ,$$

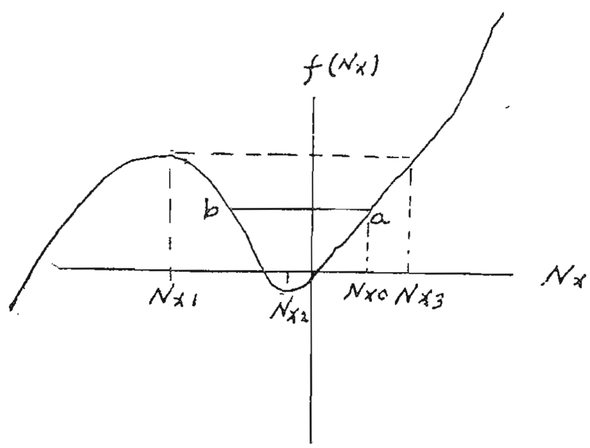
and $$f(N_x) = N_x^3 + N_x^2 X_o - N_x N_y^2 D$$

where $A = 3\,A_{11} + A_{33}$.

The constant of the motion is obtained from the initial conditions N_{xo} and $\phi_x = \psi_x = \theta = 0$. As Q_{33}, Q_{11} vary with the particle motion in the lattice N_x is determined by the constant C. When $\cos Q_{33} = \cos Q_{11} = -1$, the particle has its maximum displacement in the opposite direction from the starting displacement. This displacement is obtained by finding the negative solution of the equation

$$f(N_x) = f(N_{xo}).$$

The functional form of $f(N_x)$ for $X_o > 0$ is shown in the sketch.



One has

$$N_{x1} = -\frac{X_o}{3}(1+B)$$

$$N_{x2} = -\frac{X_o}{3}(1-B)$$

$$N_{x3} = -\frac{X_o}{3}(1-2B)$$

$$B = \sqrt{1 + \frac{3N_y{}^2 D}{X_o{}^2}}$$

For an initial condition N_{xo}, the displacement of the particle ranges between a and b as shown on the sketch. For $N_{xo} > N_{x3}$, b does not exist and the motion is unstable. The limit of stability occurs for $N_{xo} = N_{x3}$, and the displacement ranges from N_{x3} to N_{x1}.

For $D < 0$, the stable region $(N_{x3} - N_{x1})$ decreases as N_y^2 increases.

For

$$N_y^2 = N_{y\ max}^2 \equiv - \frac{X_o^2}{3D} ,$$

the stable region becomes 0. For $N_y > N_{y\ max}$, there is no stability. (For $D > 0$, the stable region increases as N_y increases.)

Figure 1 shows the comparison between the predicted stability region and the results of tracking for the 7-GeV Advanced Photon Source CDR lattice.(1) The tracking results are obtained by searching for the limit of stability in N_y for a fixed value for N_x. (Negative N_x is understood to mean $\cos \phi_{xo} = -1$.)

The limits of the stable region for this lattice are caused by the chromaticity correcting sextupoles and the predominance of the integral resonance for $\nu_x = 0.88$ per period. The values used in predicting the results are

$$\begin{aligned} \varepsilon &= 8.08 \times 10^{-9} \text{ m} \\ A_{33} &= -1.5200 \text{ m}^{-1/2} \\ A_{11} &= -1.3445 \text{ m}^{-1/2} \\ B_{11} &= 1.4385 \text{ m}^{-1/2} \end{aligned}$$

Reference

(1) 7-GeV Advanced Advanced Photon Source Conceptual Design Report ANL-87-15 (April 1987).

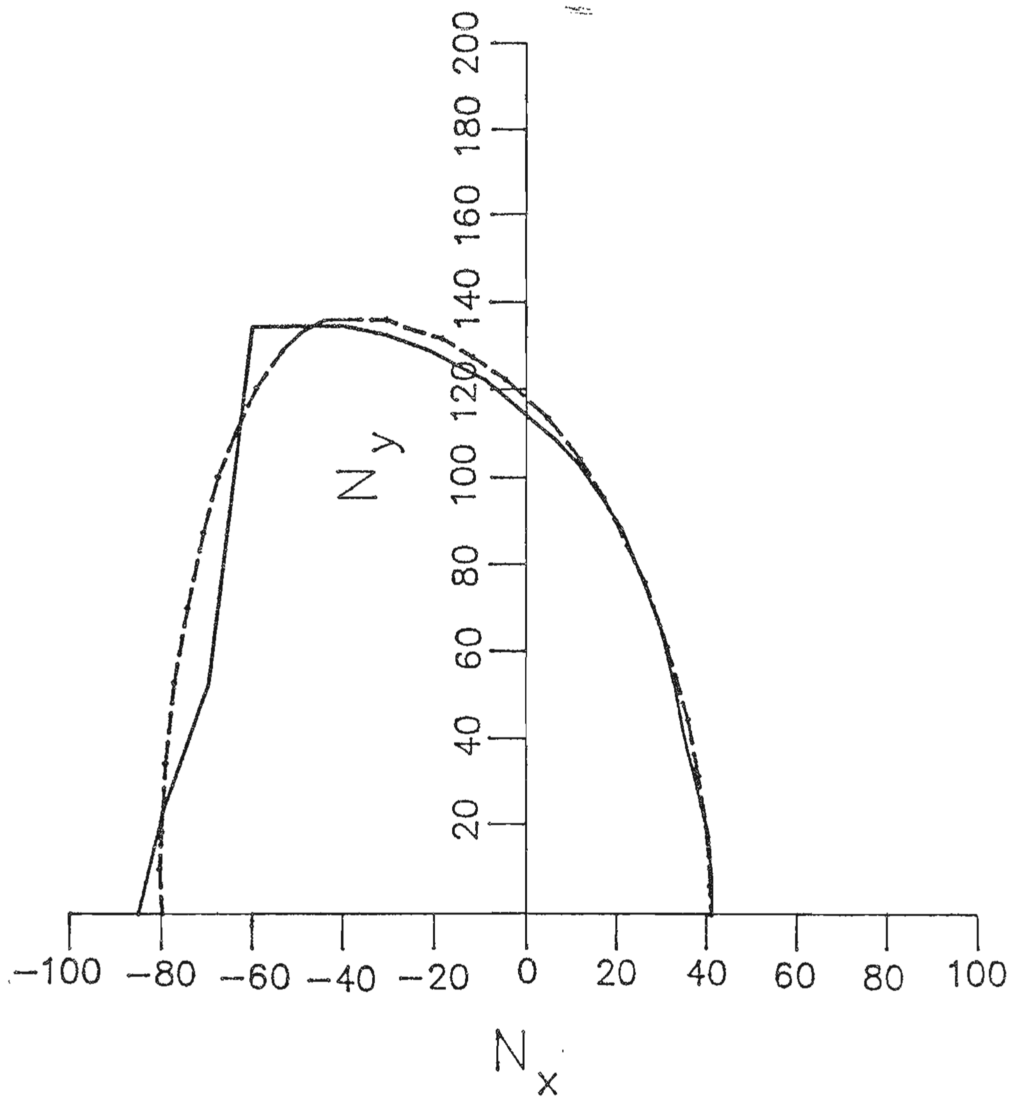


Figure 1

Dynamic aperture obtained by tracking (solid curve) and predicted by first-order resonance (dotted curve) with chromaticity-correcting sextupoles only.